A Fully-Coupled Approach for Modelling Plastic Deformation and Liquid Lubrication in Metal Forming

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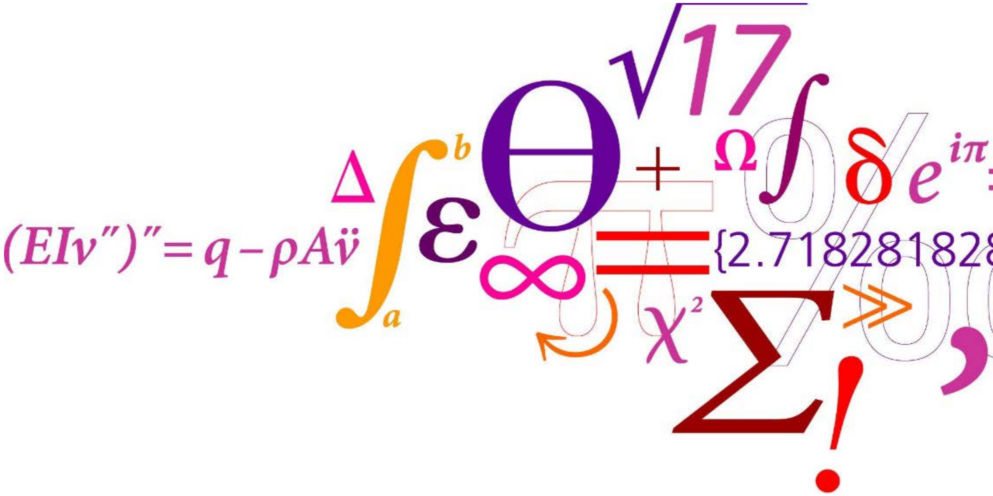

DTU Mechanical Engineering
Department of Mechanical Engineering

Outline

- Theoretical & Numerical Background
- Results & Discussion
- Summary

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- **Theoretical & Numerical Background**
    - –Equilibrium Equations
    - –FE Discretization
    - –Viscous Flow within Flow Formulation
- Results & Discussion
- Summary

Equilibrium Equation

In fluid mechanics differential equation of motion is

$$\underbrace{\frac{\partial \sigma_{ij}^{'}}{\partial x_j}}_{\text{viscous forces}} - \underbrace{\frac{\partial p}{\partial x_i}}_{\text{pressure}} + \underbrace{\rho g_i}_{\text{gravity}} = \underbrace{\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right)}_{\text{inertia term}}$$

where $\sigma_{ij} = \sigma_{ij}^{'} - p$ and $p = -\sigma_m$.

For Newtonian fluid,

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \sigma_{xx}^{'} = 2\mu\frac{\partial u}{\partial x}$$

Equilibrium Equation

If the space-dependent acceleration term of Navier-Stokes equation is neglected, the equation reduces to

$$\mu\frac{\partial^2 u_i}{\partial x_j^2}-\frac{\partial p}{\partial x_i}+\rho g_i=\rho\frac{du_i}{dt}$$

$$\sigma_{ij}=\sigma_{ij}^{'}+\sigma_m=\sigma_{ij}^{'}-p$$

which gives the dynamic equilibrium equation of solid mechanics that is used in the flow formulation.

$$\frac{\partial\sigma_{ij}}{\partial x_j}+b_i=\rho a_i \qquad b_i=\rho g_i$$

$$a_i=\frac{Du_i}{Dt}=\dot{u}_i+(u_j\cdot\nabla)u_i\cong\dot{u}_i$$

The Flow Formulation

Similarities of viscous flow with flow formulation

- viscous flow formulation

$$\mu\frac{\partial^2 u_i}{\partial x_j^2}-\frac{\partial p}{\partial x_i}+\rho g_i=\rho\frac{du_i}{dt}$$

- identical separation between deviatoric and volumetric terms
- the analogy for 'shear viscosity' and 'bulk viscosity' to ensure the incompressibility constraint of the viscous fluids.

$$\dot{\rho}+\nabla(\rho\mathbf{u})=0 \quad \overset{\partial\rho/\partial t\cong 0}{\longrightarrow} \quad \nabla\mathbf{u}=0 \quad \rightarrow \quad \dot{\varepsilon}_v=0$$

$$\sigma_m=K\dot{\varepsilon}_v \qquad \sigma_m=-p=\mu_v\,\dot{\varepsilon}_v$$

$$\sigma'_{ij}=\frac{2}{3}\frac{\overline{\sigma}}{\dot{\overline{\varepsilon}}}\dot{\varepsilon}'_{ij} \qquad \sigma'_{ij}=2\mu\,\dot{\varepsilon}'_{ij}$$

Finite Element Formulation

Weak Form of Equilibrium Equation

$$\delta\Pi = \int_V \sigma_{ij}^{'} \delta \dot{\varepsilon}_{ij}^{'} dV + K \int_V \dot{\varepsilon}_V \delta \dot{\varepsilon}_V dV - \int_{S_T} t_i \delta u_i dS = 0$$

$$\sum_{m=1}^{M} \left\{ \int_{V^m} \delta \mathbf{v}^T K_\mu \, \mathbf{v} \, dV^m + K^m \int_{V^m} \delta \mathbf{v}^T \mathbf{C}^T \mathbf{B} \mathbf{v} \mathbf{C}^T \mathbf{B} \, dV^m - \int_{S_T^m} \delta \mathbf{v}^T \mathbf{N} \mathbf{T} dS^m \right\} = 0$$

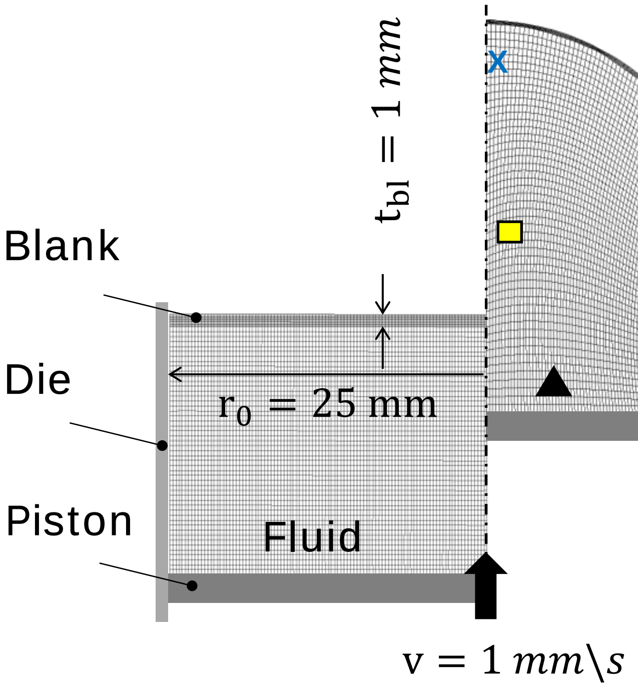
$$\mathbf{K}_\mu = \mathbf{B}^{\mathbf{T}} \mathbf{D}_\mu \mathbf{B} \qquad \mathbf{C}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \mathbf{L}\,\mathbf{N}^T \qquad K^m = \mu_v^m = k_0 - k_1 \cdot \frac{\sigma_{ii}}{3}$$

$$\mathbf{D}_\mu = \begin{bmatrix} 2\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \qquad \mathbf{N}^{\mathbf{T}} = \begin{bmatrix} N_1 & 0 & 0 & \ldots & N_8 & 0 & 0 \\ 0 & N_1 & 0 & \ldots & 0 & N_8 & 0 \\ 0 & 0 & N_1 & \ldots & 0 & 0 & N_8 \end{bmatrix}$$

Outline

- Theoretical & Numerical Background
- **Results & Discussion**
  - Bulge Test
  - Lubrication Breakdown
  - Drawing of Sheet with Surface Pockets
  - Deep Drawing
- Summary

Bulge Test



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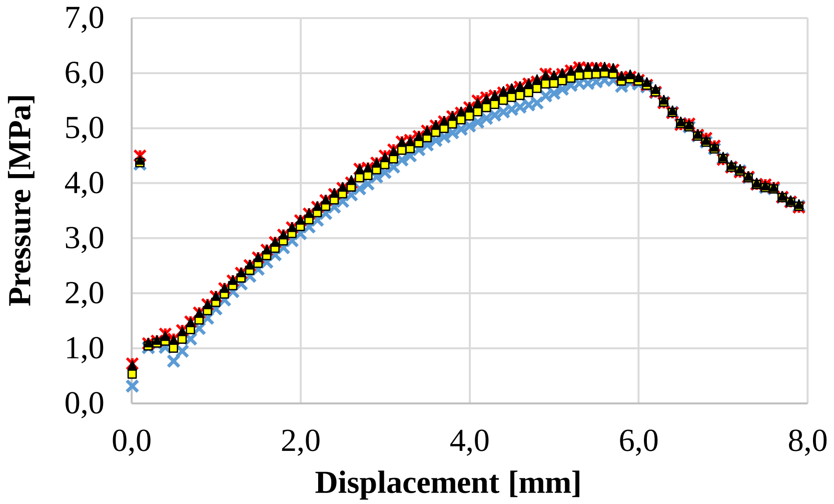
$$\bar{\sigma} = 140\bar{\varepsilon}^{0.04}\ (MPa)$$

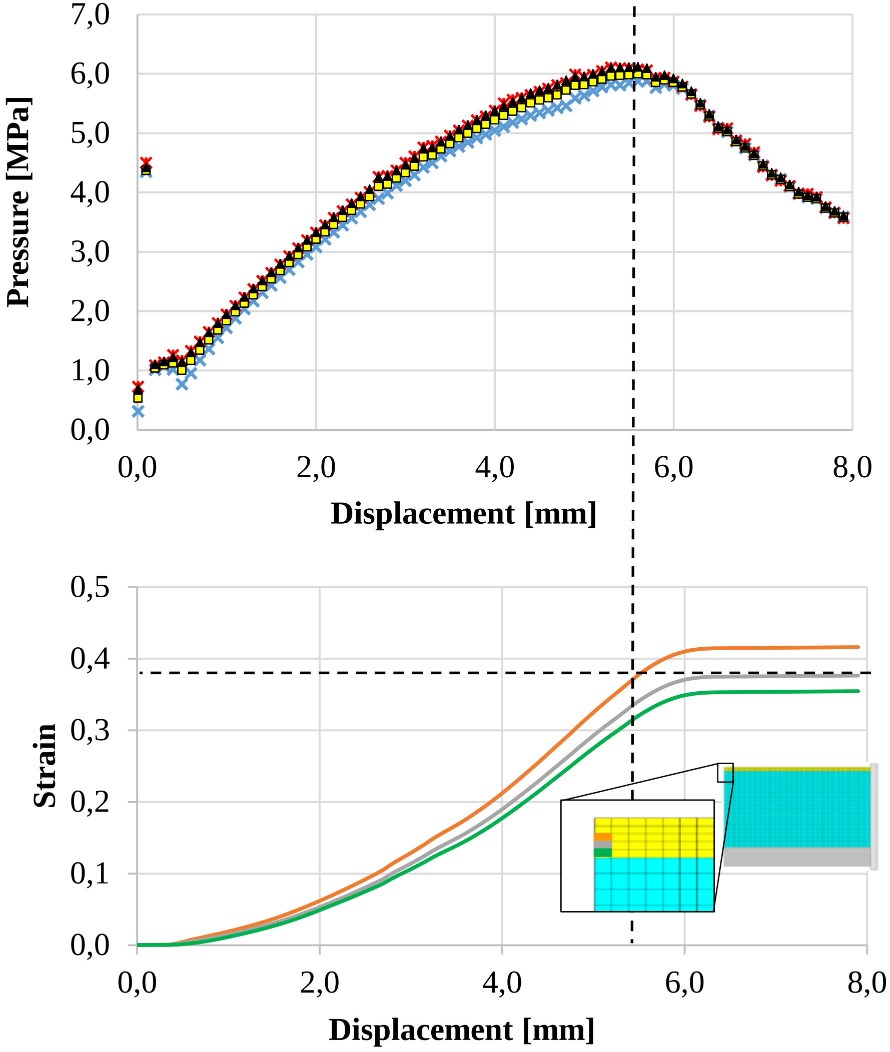
Fluid:

$$\mu = 10^2\ Pa \cdot s$$

$$\mu_v = 10^3\ MPa \cdot s$$



Bulge Test



uniform distribution of hydrostatic pressure!

onset of instability (Hill, 1950)

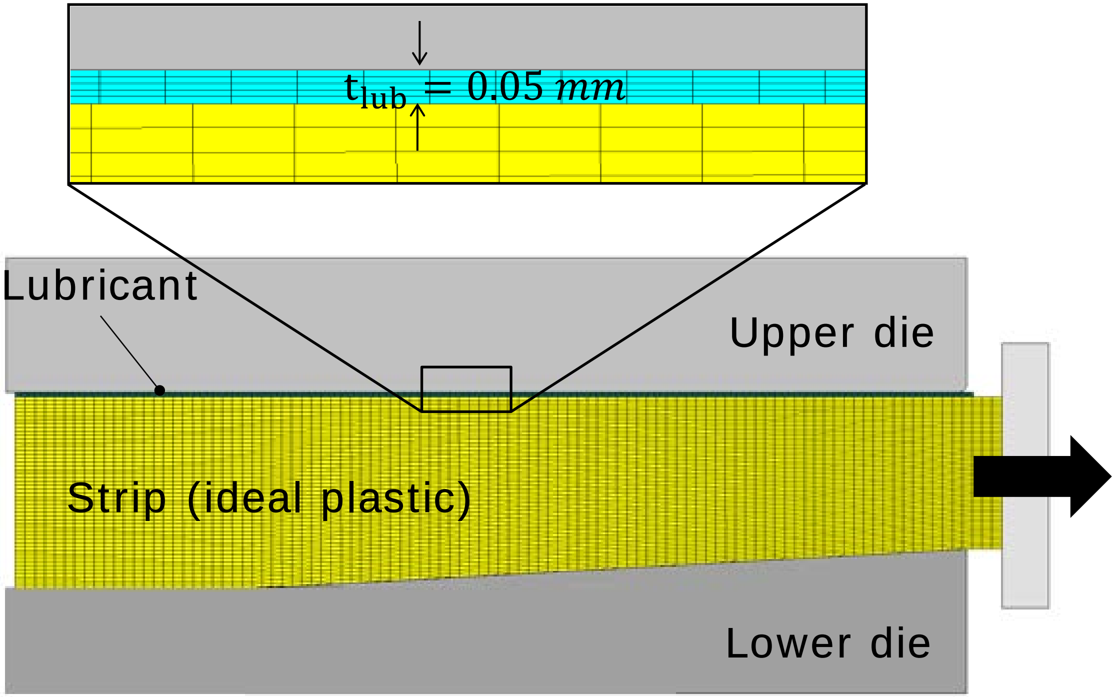
$$\bar{\varepsilon}_{inst,cal} = \frac{4}{11}(2n+1)$$

$$\bar{\varepsilon}_{inst,cal} = 0.39$$

$$\bar{\varepsilon}_{inst,num} = 0.37$$

$$error = 2\ \%$$

Lubricant Breakdown



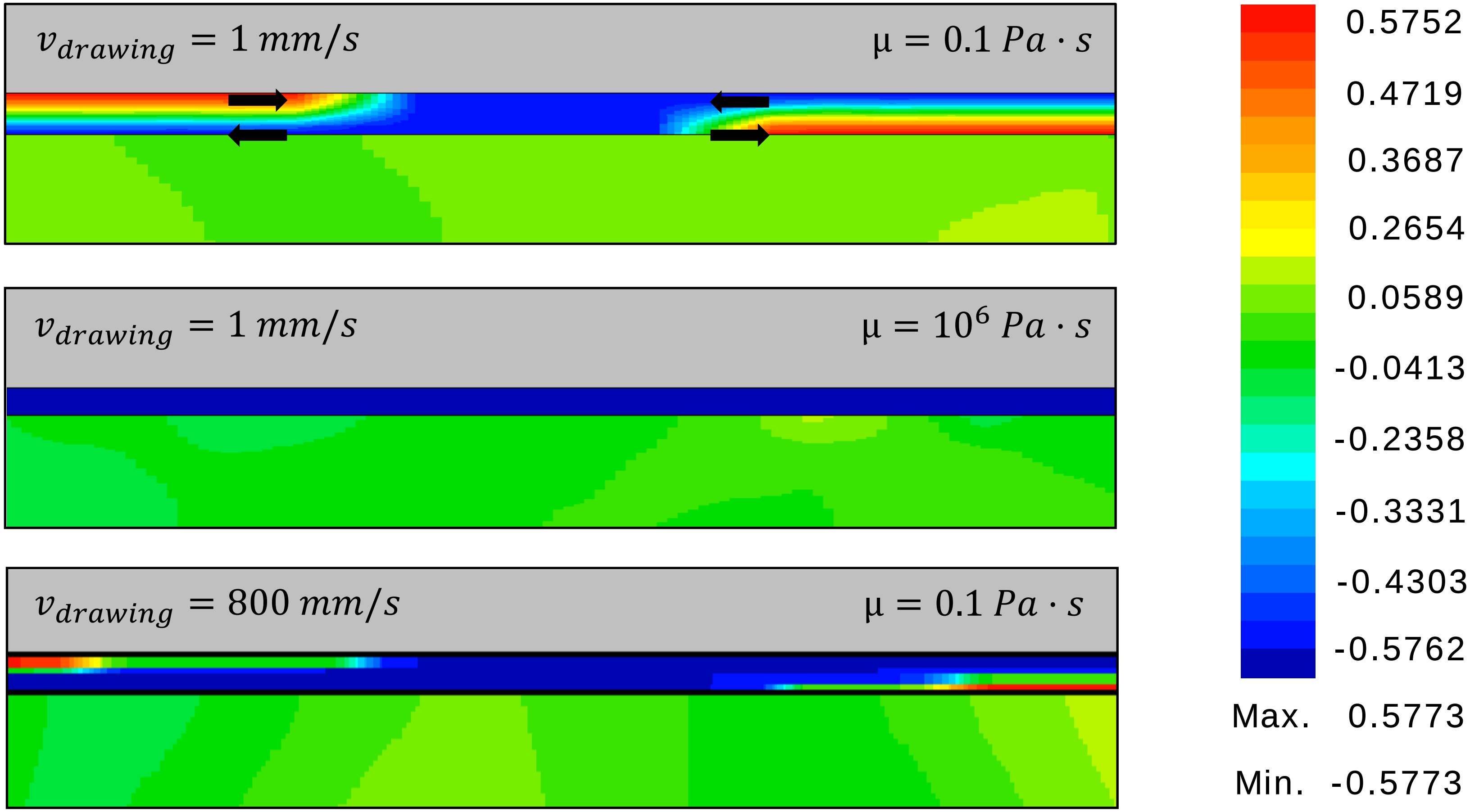
$\mathrm{m} = 1.0$ sticking interfaces

$\alpha_{lover\ die} = 3.0°$

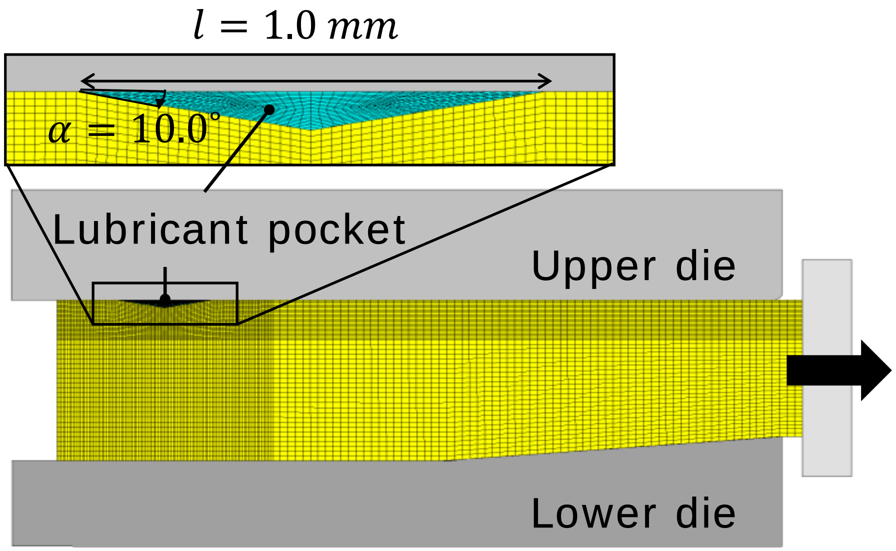
$h_{strip} = 1.95\,mm$

Lubricant Breakdown

Shear Stress Normalized by Effective Stress



Drawing of Sheet with Surface Pocket



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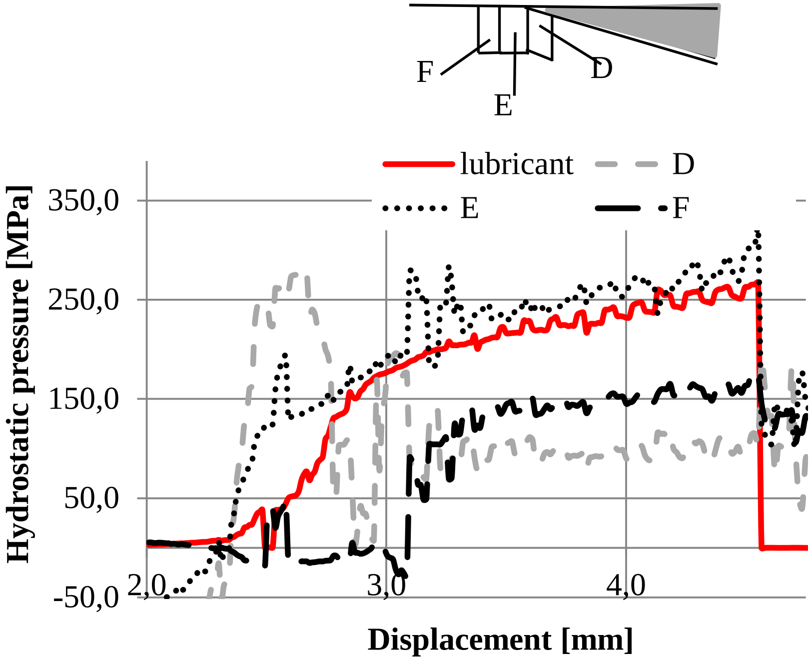
$$\bar{\sigma} = 312\bar{\varepsilon}^{0.08}\ (MPa)$$

Fluid: $\mu = 0.1\ Pa \cdot s$



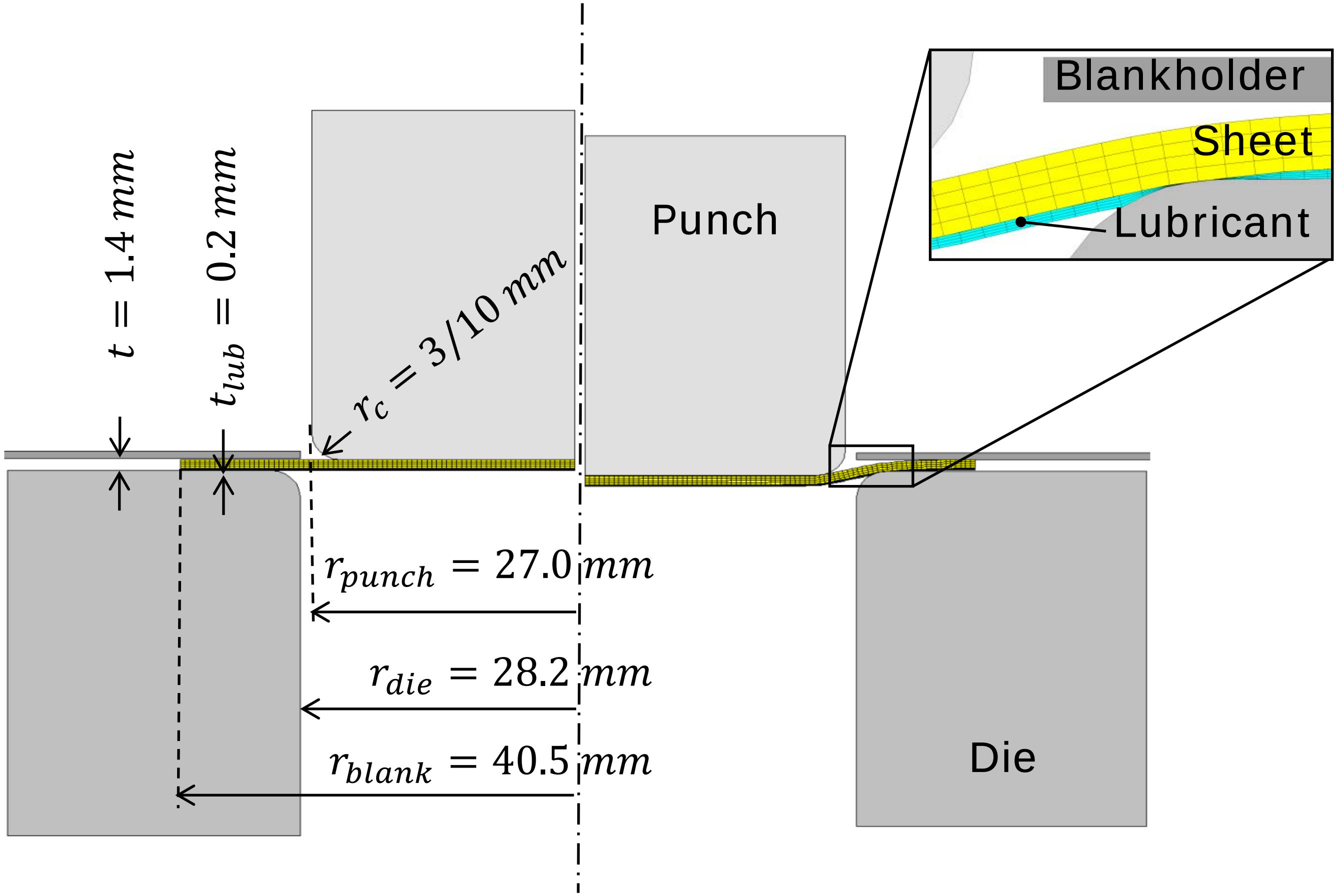
$v_{drawing} = 100\ mm/s$

$\alpha_{lover\ die} = 5.0°$

$m = 0.3$ along the upper tool

$m = 1.0$ sheet – lubricant interface

Deep Drawing



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Fluid:	$\mu = 10^6\ Pa \cdot s$

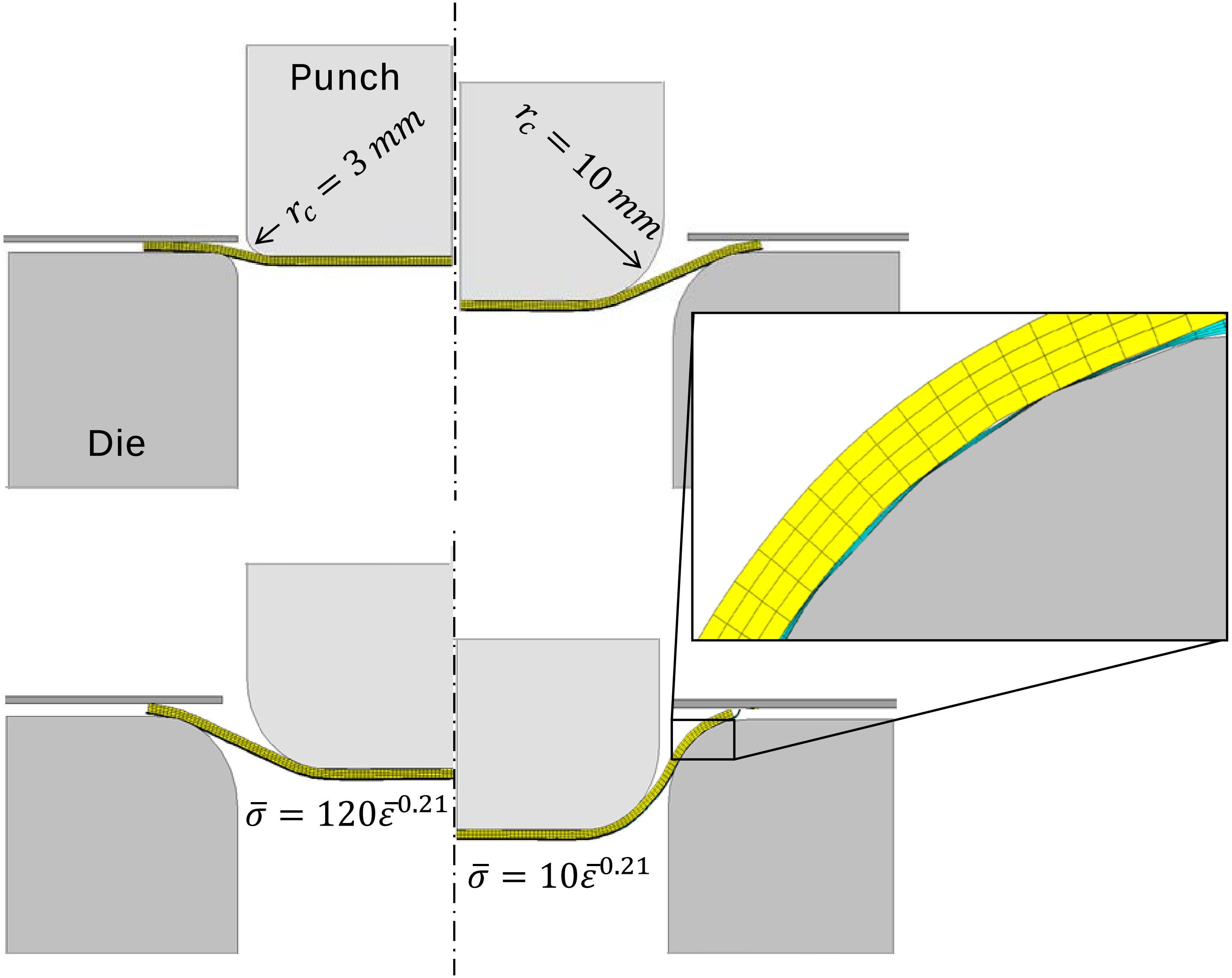
$\mathrm{m} = 0.5$ sheet – lubricant interface

$v_{drawing} = 1\ mm/s$

Deep Drawing

$\bar{\sigma} = 120\bar{\varepsilon}^{0.21}$

$r_c = 10\ mm$



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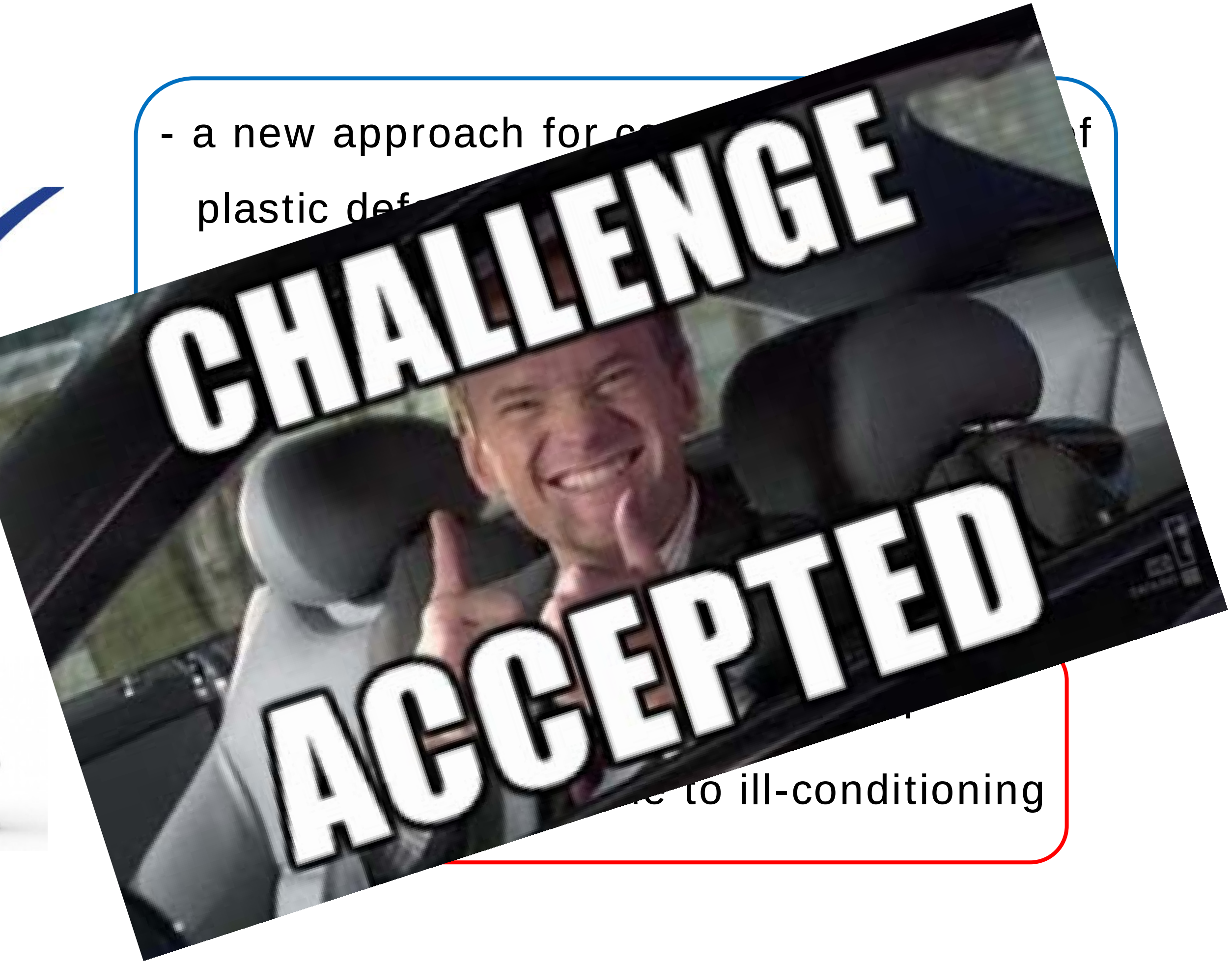
  - Current State
  - Challenges
  - Future Work

Summary

- a new approach for c f

plastic def



e to ill-conditioning

Future Works

decoupling of the two models simulating metal flow and liquid flow separately

Back-Up

The Flow Formulation

- The weak form of the quasi static stress equilibrium equations used in the flow formulation results from neglecting gravity and dynamic effects.
- Incompressibility is included by transforming the general weak form of the stress equilibrium equations into a weak form with constraints.
- Flow formulation is based on a velocity-pressure approach where metals are considered as fluids of very high viscosity and velocities and hydrostatic pressures are solved simultaneously.

Drawing of Sheet with Surface Pocket

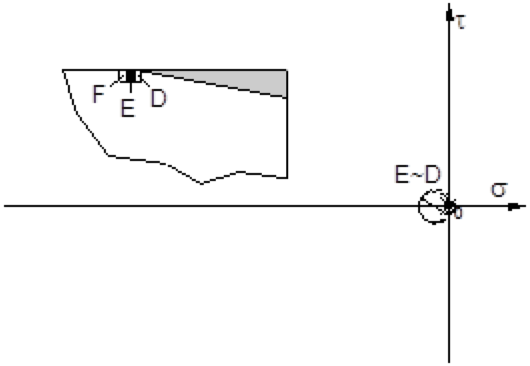
Back of the Pocket

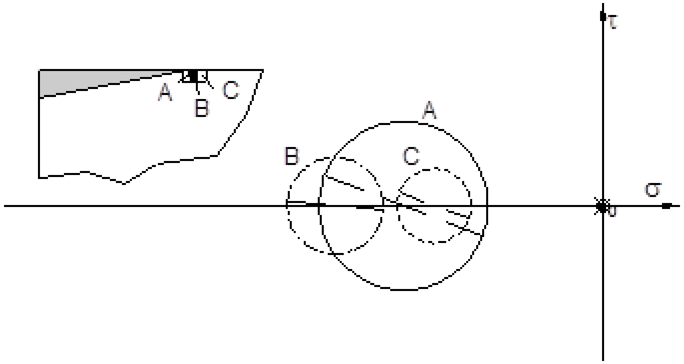
$v_{drawing} = 100\, mm/s$

Front of the Pocket

$v_{drawing} = 1\, mm/s$

Beginning of the Process





Later Stage